EXERCISE – I SINGLE CORRECT (OBJECTIVE QUESTIONS)

1. The interval in which the function x^3 increases less rapidly than $6x^2 + 15x + 5$ is
(A) $(-\infty, -1)$ (B) $(-5, 1)$ (C) $(-1, 5)$ (D) $(5, \infty)$

2. The function $\dfrac{|x-1|}{x^2}$ is monotonically decreasing in
(A) $(2, \infty)$ (B) $(0, 1)$
(C) $(0, 1)$ and $(2, \infty)$ (D) $(-\infty, \infty)$

3. If $y = (a + 2)x^3 - 3ax^2 + 9ax - 1$ decreases monotonically $\forall\ x \in R$ then 'a' lies in the interval
(A) $(-\infty, -3]$ (B) $(-\infty, -2) \cup (-2, 3)$
(C) $(-3, \infty)$ (D) None of these

4. The values of p for which the function

$f(x)=\left(\dfrac{\sqrt{p+4}}{1-p}-1\right)x^5 - 3x + \ln 5$ decreases for all real x is

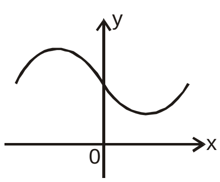
(A) $(-\infty, \infty)$ (B) $\left[-4, \dfrac{3-\sqrt{21}}{2}\right] \cup (1, \infty)$

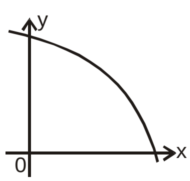
(C) $\left[-3, \dfrac{5-\sqrt{27}}{2}\right] \cup (2, \infty)$ (D) $(1, \infty)$

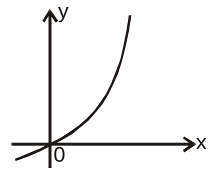
5. The true set of real values of x for which the function, $f(x) = x\,ln\,x - x + 1$ is positive is
(A) $(1, \infty)$ (B) $(1/e, \infty)$
(C) $[e, \infty)$ (D) $(0, 1)$ and $(1, \infty)$

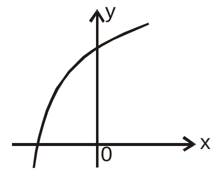
6. The set of all x for which $ln\,(1 + x) \le x$ is equal to
(A) $x > 0$ (B) $x > -1$ (C) $-1 < x < 0$ (D) null set

7. The curve $y = f(x)$ which satisfies the condition $f'(x) > 0$ and $f''(x) < 0$ for all real x, is

(A) 

(B) 

(C) 

(D) 

8. For which values of 'a' will the function

$f(x) = x^4 + ax^3 + \dfrac{3x^2}{2} + 1$ will be concave upward along the entire real line
(A) $a \in [0, \infty)$ (B) $a \in (-2, 2)$
(C) $a \in [-2, 2]$ (D) $a \in (0, \infty)$

9. If the point (1, 3) serves as the point of inflection of the curve $y = ax^3 + bx^2$ then the value of 'a' and 'b' are
(A) $a = 3/2$ & $b = -9/2$ (B) $a = 3/2$ & $b = 9/2$
(C) $a = -3/2$ & $b = -9/2$ (D) $a = -3/2$ & $b = 9/2$

10. The function $f(x) = x^3 - 6x^2 + ax + b$ satisfy the conditions of Rolle's theorem in [1, 3]. The value of a and b are
(A) 11, –6 (B) –6, 11 (C) –11, 6 (D) 6, –11

11. The function $f(x) = x(x + 3)\,e^{-x/2}$ satisfies all the conditions of Rolle's theorem in [–3, 0]. The value of c which verifies Rolle's theorem, is
(A) 0 (B) –1 (C) –2 (D) 3

12. If $f(x) = a^{\{a^{|x|}\operatorname{sgn} x\}}$; $g(x) = a^{[a^{|x|}\operatorname{sgn} x]}$ for $a > 1, a \ne 1$ and $x \in R$, where $\{*\}$ & $[*]$ denote the fractional part and integral part functions respectively, then which of the following statements holds good for the function h(x), where $(ln\,a)\,h(x) = (ln\,f(x) + ln\,g(x))$.
(A) 'h' is even and increasing
(B) 'h' is odd and decreasing
(C) 'h' is even and decreasing
(D) 'h' is odd and increasing

13. Let $f(x) = (x - 4)(x - 5)(x - 6)(x - 7)$ then,
(A) $f'(x) = 0$ has four roots
(B) Three roots of $f'(x) = 0$ lie in $(4, 5) \cup (5, 6) \cup (6, 7)$
(C) The equation $f'(x) = 0$ has only one real root
(D) Three roots of $f'(x) = 0$ lie in $(3, 4) \cup (4, 5) \cup (5, 6)$

14. For what values of a does the curve $f(x) = x(a^2 - 2a - 2) + \cos x$ is always strictly monotonic $\forall$ $x \in R$.
(A) $a \in R$ (B) $|a| < \sqrt{2}$
(C) $1-\sqrt{2} \le a \le 1+\sqrt{2}$ (D) $|a| < \sqrt{2} - 1$

15. Given that f is a real valued differentiable function such that $f(x)\, f'(x) < 0$ for all real x, it follows that
(A) f(x) is an increasing function
(B) f(x) is a decreasing function
(C) |f(x)| is an increasing function
(D) |f(x)| is a decreasing function

16. If $f(x) = \dfrac{x^2}{2-2\cos x}$; $g(x) = \dfrac{x^2}{6x - 6\sin x}$ where $0 < x < 1$, then
(A) both 'f' and 'g' are increasing functions
(B) 'f' is decreasing & 'g' is increasing function
(C) 'f' is increasing & 'g' is decreasing function
(D) both 'f' & 'g' are decreasing function

17. If the function $f(x) = x^3 - 6ax^2 + 5x$ satisfies the conditions of Lagrange's mean theorem for the interval [1, 2] and the tangent to the curve $y = f(x)$ at $x = 7/4$ is parallel to the chord joining the points of intersection of the curve with the ordinates $x = 1$ and $x = 2$. Then the value of a is
(A) 35/16 (B) 35/48 (C) 7/16 (D) 5/16

18. $f : R \to R$ be a differentiable function $\forall$ $x \in R$. If tangent drawn to the curve at any point $x \in (a, b)$ always lie below the curve, then
(A) $f'(x) > 0$ $f''(x) < 0$ $\forall$ $x \in (a, b)$
(B) $f'(x) < 0$ $f''(x) < 0$ $\forall$ $x \in (a, b)$
(C) $f'(x) > 0$ $f''(x) > 0$ $\forall$ $x \in (a, b)$
(D) None of these

19. A value of C for which the conclusion of Mean Value Theorem holds for the function $f(x) = \log_e x$ on the interval [1, 3] is
(A) $2 \log_3 e$ (B) $\frac{1}{2}\log_e 3$ (C) $\log_3 e$ (D) $\log_e 3$

20. The function $f(x) = \tan^{-1}(\sin x + \cos x)$ is an increasing function in
(A) $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ (B) $\left(-\frac{\pi}{2}, \frac{\pi}{4}\right)$ (C) $\left(0, \frac{\pi}{2}\right)$ (D) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

21. A function is matched below against an interval where it is supposed to be increasing. Which of the following pairs is incorrectly matched ?

	Interval	**Function**
(A)	$(-\infty, -4]$	$x^3 + 6x^2 + 6$
(B)	$\left(-\infty, \frac{1}{3}\right]$	$3x^3 - 2x + 1$
(C)	$[2, \infty)$	$2x^3 - 3x^2 - 12x + 6$
(D)	$(-\infty, \infty)$	$x^3 - 3x^2 + 3x + 3$

22. A function $y = f(x)$ has a second order derivative $f'' = 6(x - 1)$. If its graph passes through the point (2, 1) and at that point the tangent of the graph is $y = 3x - 5$, then the function is
(A) $(x-1)^2$ (B) $(x-1)^3$ (C) $(x+1)^3$ (D) $(x+1)^2$

23. If $f(x) = [a \sin x + b \cos x] / [c \sin x + d \cos x]$ is monotonically increasing, then
(A) $ad \ge bc$ (B) $ad < bc$ (C) $ad \le bc$ (D) $ad > bc$

24. $x^3 - 3x^2 - 9x + 20$ is
(A) –ve for $x < 4$ (B) +ve for $x > 4$
(C) –ve for $x \in (0, 1)$ (D) –ve for $x \in (-1, 0)$

25. $f(x) = x^2 - x \sin x$ is
(A) $\uparrow$ for $0 \le x \le \pi/2$ (B) $\downarrow$ for $0 \le x \le \pi/2$
(C) $\downarrow$ for $[\pi/4, \pi/2]$ (D) None of these

26. The number of values of 'c' of Lagrange's mean value theorem for the function,
$f(x) = (x - 1)(x - 2)(x - 3)$, $x \in (0, 4)$ is
(A) 1 (B) 2 (C) 3 (D) None of these

27. The equation $xe^x = 2$ has
(A) one root of $x < 0$ (B) two roots for $x > 1$
(C) no root in (0, 1) (D) one root in (0, 1)

28. If $f(x) = 1 + x\, ln\left[x + \sqrt{x^2+1}\right]$ and $g(x) = \sqrt{x^2+1}$ then for $x \ge 0$
(A) $f(x) < g(x)$ (B) $f(x) > g(x)$
(C) $f(x) \le g(x)$ (D) $f(x) \ge g(x)$

29. The set of values of the parameter 'a' for which the function; $f(x) = 8ax - a \sin 6x - 7x - \sin 5x$ increases & has no critical points for all $x \in R$, is
(A) $[-1, 1]$ (B) $(-\infty, -6)$ (C) $(6, +\infty)$ (D) $[6, +\infty)$

30. If f(x) and g(x) are differentiable in [0, 1] such that f(0) = 2, g(0) = 0, f(1) = 6, g(1) = 2, then Rolle's theorem is applicable for which of the following
(A) f(x) – g(x) (B) f(x) – 2g(x)
(C) f(x) + 3g(x) (D) None of these

31. $f : [0, 4] \to R$ is a differentiable function then for some a, b $\in$ (0, 4), $f^2(4) - f^2(0)$ equals
(A) 8f′(a) . f(b) (B) 4f′(a) f(b)
(C) 2f′(a) f(b) (D) f′(a) f(b)

32. Equation $3x^2 + 4ax + b = 0$ has at least one root in (0, 1) if
(A) 4a + b + 3 = 0 (B) 2a + b + 1 = 0
(C) $b = 0, a = -\frac{4}{3}$ (D) None of these

33. If $0 < a < b < \frac{\pi}{2}$ and $f(a, b) = \frac{\tan b - \tan a}{b - a}$, then
(A) $f(a, b) \geq 2$ (B) $f(a, b) \geq 1$
(C) $f(a, b) \leq 1$ (D) None of these

34. Let $f(x) = ax^4 + bx^3 + x^2 + x - 1$. If $9b^2 < 24a$, then number of real roots of f(x) = 0 are
(A) 4 (B) > 2 (C) 0 (D) can't say

35. Function for which LMVT is applicable but Rolle's theorem is not
(A) $f(x) = x^3 - x, x \in [0, 1]$
(B) $f(x) = \begin{cases} x^2 & , \quad 0 \leq x < 1 \\ x & , \quad 1 < x \leq 2 \end{cases}$
(C) $f(x) = e^x, x \in [-3, 3]$
(D) $f(x) = 1 - \sqrt[3]{x^2}, x \in [-1, 1]$

36. LMVT is not applicable for which of the following ?
(A) $f(x) = x^2, x \in [3, 4]$ (B) $f(x) = \ln x, x \in [1, 3]$
(C) $f(x) = 4x^2 - 5x^2 + x - 2, x \in [0, 1]$
(D) $f(x) = \{x^4(x-1)\}^{1/5}, x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

37. If f(x) = (x – 1) (x – 2) (x – 3) (x – 4), then roots of f′(x) = 0 not lying in the interval
(A) [1, 2] (B) (2, 3) (C) (3, 4) (D) $(4, \infty)$

38. If $f(x) = 1 + x^m (x - 1)^n$, m, n $\in$ N, then f′(x) = 0 has atleast one root in the interval
(A) (0, 1) (B) (2, 3) (C) (–1, 0) (D) None of these